Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

371. Proposed by ELMER SCHUYLER, Brooklyn, N. Y.

In a G. P. of an odd number of terms, all of the terms being positive, and the ratio different from 1, show that the middle term is less than the arithmetical mean.

I. Solution by ELIJAH SWIFT, Princeton University.

We have a G. P. of an odd number of positive terms, a, ar, ar^2, ..., ar^{2n}.

To prove $ar^n < \frac{a(r^{2n+1}-1)}{(2n+1)(r-1)}$.

Assuming $r>1$ and clearing, we have to prove, a being positive,

$$(2n+1)(r^{n+1}-r^n) < r^{2n+1}-1; \text{ or}$$

$$r^{2n+1}-(2n+1)r^{n+1}+(2n+1)r^n-1>0).$$

Call the expansion on the left $f(r)$. Then $f(1)=0$.

$$f'(r)=(2n+1)r^{n-1}\{r^{n+1}-(n+1)r+n\}.$$

Call the expression in brackets $\phi(r)$. Then $\phi(1)=0$.

$$\phi'(r)=(n+1)\{r^n-1\}.$$

Then we have, since $r>1$, $n>0$, $\phi'(r)>0$.

Since $\phi(1)=0$ and $\phi'(r)>0$, $\phi(r)>0$. But $f'(r)=(2n+1)r^{n-1}.\phi(r)$. Therefore $f'(r)>0$, and finally, $f(r)>0$.

The case where $r<1$ may be treated by reversing the inequality signs.

Also solved by H. Prime, H. C. Feemster, C. E. Githens, A. M. Harding, and the Proposer.

372. Proposed by S. LEFSCHETZ, Ph. D., University of Nebraska.

Prove that $\sum_{n=1}^{n=\infty} n^2x^{n-1}=\frac{1+x}{(1-x)^3}$, if mod.$x<1$. (*Schlömilch.*)

Solution by **H. C. FEEMSTER**, York College, York, Nebraska.

$$\sum_{n=1}^{n=\infty} n^2x^{n-1}=1+4x+9x^2+16x^3+\ldots+r^2x^{r-1}+(r+1)^2x^r+(r+2)^2x^{r+1}+\ldots.$$

and
$$(1-x)^3\sum_{n=1}^{n=\infty} n^2x^{n-1}=[1+4x+9x^2+16x^3+\ldots+r^2x^{r-1}+(r+1)^2x^r$$
$$+(r+2)^2x^{r+1}+\ldots]-[3x+12x^2+27x^3+\ldots+3(r-1)^2x^{r-1}+3r^2x^r$$
$$+3(r+1)^2x^{r+1}+\ldots+[3x^2+12x^3+\ldots+3(r-2)^2x^{r-1}+3(r-1)^2x^r$$
$$+3r^2x^{r+1}+\ldots-[x^3+\ldots+(r-3)^2x^{r-1}+(r-2)^2x^r+(r-1)^2x^{r+1}+\ldots]$$
$$=1+x, \text{ so}$$

$$\therefore \sum_{n=1}^{n=\infty} n^2x^{n-1}=\frac{1+x}{(1-x)^3}, \text{ as required.}$$

Also solved by H. Prime, J. Scheffer, A. M. Harding, and Elijah Swift.

373. Proposed by **X**, National Electric Light Association, Brooklyn, N. Y.

(a) For underground distribution of direct current electrical energy, we have $DA^n-CB^n=H$, where the only unknown is n, which represents the number of years it will take a large direct current low tension feeder to pay by line loss saving for the increased investment over a smaller feeder.

(b) $bVf^2l^2\beta^2+aVf\beta^{1.6}=W$, the iron loss equation which is to be solved for β. When $bVf^2l^2\beta^2$ represents the eddy current loss in the core of a transformer, and $aVf\beta^{1.6}$is the hysteresis loss in the core. a, b, and l are constants of the core, V is the voltage, f is the frequency, and β is the flux density and is the only unknown in the equation.

Letting $bVf^2l^2=A$, and $aVf=C$, we have $A\beta^2+C\beta^{1.6}=W$.

Solution by **E. B. ESCOTT**, Ann Arbor, Michigan.

(a) $DA^n-CB^n=H$, n unknown. Since $A=10^{\log A}$ and $B=10^{\log B}$, the equation may be written

$$D.10^{n\log A}-C.10^{n\log B}=H.$$

Putting $10^n=x$, the equation becomes $Dx^{\log A}-Cx^{\log B}=H$.

This equation and the one under

$$\text{(b)} \qquad A\beta^2+C\beta^{1.6}=W$$

are trinomial equations, which may be solved in various ways. One of the simplest methods to use in practice is by addition and subtraction of logarithms. This is given in C. Runge, *Praxis der Gleichungen* (Leipzig, 1900), pages 140-157.

Gundelfinger has given tables to three decimals which enable one to